Chapter 4 "Performance Analysis of Parallel Programs"

according to the book
"Parallel Programming"
by T. Rauber and G. Rünger
Springer 2010

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Lecture outline

1. Performance Evaluation of Computer Systems

2. Performance for Parallel Programs

3. Implementation of global communication operations

Evaluation of CPU Performance

The response time of a program A can be split into

- the **user CPU time** of A, capturing the time that the CPU spends for executing A;
- the **system CPU time** of A, capturing the time that the CPU spends for the execution of routines of the operating system issued by A;
- the **waiting time** of A, caused by waiting for the completion of I/O operations and by the execution of other programs because of time sharing.

In the following, we concentrate on the **user CPU time**.

MIPS as performance measure

- A performance measure often used in practice to evaluate the performance of a computer system is the **MIPS rate** for a program A:

$$MIPS(A) = \frac{n_{instr}(A)}{T_{U\_CPU}(A) \cdot 10^6} \quad . \tag{1}$$

  $n_{instr}(A)$: number of instructions of program A
  $T_{U\_CPU}(A)$: user CPU time of program A

- modification:

$$MIPS(A) = \frac{r_{cycle}}{CPI(A) \cdot 10^6} \ ,$$

  where $r_{cycle} = 1/t_{cycle}$ is the clock rate of the processor.
  $CPI(A)$: **C**lock cycles **P**er **I**nstruction: average number of CPU cycles used for instructions of program A

- Faster processors lead to larger MIPS rates than slower processors.

MFLOPS as performance measure

- For program with scientific computations, the MFLOPS rate (**M**illion **F**loating-point **O**perations **P**er **S**econd) is sometimes used. The MFLOPS rate of a program A is defined by

$$MFLOPS(A) = \frac{n_{flp\_op}(A)}{T_{U\_CPU}(A) \cdot 10^6} \; [1/s] \; , \tag{2}$$

  $n_{flp\_op}(A)$: number of floating-point operations executed by A.
  $T_{U\_CPU}(A)$: user CPU time of program A

- The effective number of operations performed is used for MFLOPS: the MFLOPS rate provides a fair comparison of different program versions performing the same operations.

Benchmark Programs

Different benchmark programs have been proposed for the evaluation of computer systems:

- **Synthetic benchmarks**
- **Kernel benchmarks**: small but relevant parts of real applications
- **Real application benchmarks** comprise several entire programs which reflect a workload of a standard user.
- popular benchmark suite: SPEC benchmarks (System Performance Evaluation Cooperation), see `www.spec.org`
- SPEC06 is the current version for desktop computers:
  12 integer programs (9 written in C, 3 in C++) and 17 floating-point programs (6 written in Fortran, 3 in C, 4 in C++, and 4 in mixed C and Fortran).

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Parallel Runtime of a Program

- The **parallel runtime** $T_p(n)$ of a parallel program P with input size n on p processors is the **time between the start of program** P and the **termination** of the computations of P on **all** processors.
- For computers with physically **distributed memory** $T_p(n)$ consists of:
  - Time for **local computations**
  - Time for **data exchange** with communication operations;
  - **Waiting time** of processors e.g. because of load imbalance
  - Time for **synchronization** of the executing processors or a subset of the executing processors
- For computers with shared memory the time for **data exchange** is replaced by the time for the **access to global data**.

Costs of Parallel Programs

- The **costs** $C_p(n)$ of a parallel program P with input size n on p processors is the **total time** that the participating processors require for the execution of P:

$$C_p(n) = T_p(n) \cdot p$$

- The cost of a parallel program is **a measure for all the computations performed**.
- A parallel program is **cost optimal** if $C_p(n) = T^*(n)$ holds, where $T^*(n)$ is the runtime of the fastest sequential program;
  A cost optimal program requires **as many computations** as the **fastest sequential program**.
  Difficulty: The fastest sequential program or method is possibly **not known** or can only be determined with a high efforts.
- The costs is often called **work** or **Processor-Time-Product**;

Speedup of a Parallel Program

- The **Speedup** $S_p(n)$ of a parallel program P with input size n on p processors is defined as:

$$S_p(n) = \frac{T^*(n)}{T_p(n)} \quad \frac{\text{sequential time}}{\text{parallel time}}$$

  - $T_p(n)$ = Parallel runtime of a parallel program P on p processors;
  - $T^*(n)$ = Runtime of an optimal sequential implementation for the solution of the problem;
- The speedup is a **measure of the relative speed increase compared to the best sequential implementation**
- Typically at most **linear speedup** can be reached: $S_p(n) \leq p$ (theoretical upper bound)
- In practice, due to cache effects a **super linear speedup** can occur.

Efficiency of a Parallel Program

- The **Efficiency** $E_p(n)$ of a parallel program P with input size n on p processors is defined by

$$E_p(n) = \frac{T^*(n)}{C_p(n)} = \frac{S_p(n)}{p} = \frac{T^*(n)}{p \cdot T_p(n)}$$

  - $C_p(n)$ = Parallel program cost
  - $T_p(n)$ = Parallel runtime of a parallel program P
  - $T^*(n)$ = Runtime of the best sequential implementation
- The efficiency is a **measure** for the portion of the runtime, that is required for computations that are also present in the sequential program.
- The ideal speedup $S_p(n) = p$ is equivalent to $E_p(n) = 1$.

Amdahl's Law

- When the parallel implementation requires a (constant) fraction f, $0 \leq f \leq 1$, to be computed **sequentially**, the runtime of the parallel implementation is composed of:
  - The runtime $f \cdot T^*(n)$ of the **sequential part** and
  - The runtime of the **parallel part**, which is at least $(1-f)/p \cdot T^*(n)$
- Attainable speedup

$$S_p(n) = \frac{T^*(n)}{f \cdot T^*(n) + \frac{1-f}{p} T^*(n)} = \frac{1}{f + \frac{1-f}{p}} \leq \frac{1}{f}.$$

- **Example:** $f = 0.2 \rightarrow S_p(n) \leq 5$ independent from the number p of processors;
  $\rightarrow$ The sequential part has a big influence on the attainable speedup; for an efficient utilization of a **high number of processors**, a **reduction** of the **sequential parts** is required.

Scalability

- The **scalability** of a parallel program on a parallel computer is a **measure** for the property **to get a performance increase proportional to the number p of processors**.
- Common observations:
  - For a fixed problem size n and an increasing number of processors p a **saturation of the speedup** occurs;
  - For a fixed number of processors p and an increasing problem size n, an **increase of the speedup** occurs.
- **Concretization:** Scalability means, that the efficiency of a parallel program is constant when the number of processors p and the problems size n is increased

Lecture outline

1. Performance Evaluation of Computer Systems

2. Performance for Parallel Programs

3. Implementation of global communication operations
   - Communication operations on static networks
   - Communication operations on a hypercube network

Overview

- We consider **global communication operations** and their implementation on **static interconnection networks** (array, ring, mesh, hypercube)
- How can these communication operations be efficiently **implemented** on these networks?

Assumptions for the analysis

The following analysis makes the following assumptions:

(a) The links of the network are **bidirectional**, i.e., messages can be sent simultaneously in both directions

(b) Each node can **simultaneously send** out messages on all outgoing links;
organization: use of output buffers with a separate controller for each link

(c) Each node can **simultaneously receive** messages on all incoming links;
organization: input buffer for each incoming link

(d) Each message consists of several bytes which are transmitted without any interruption

(e) The time for transmitting a message consists of
- the **startup time** t_s (independent of the message size)
- the **transfer time** $U = n \cdot t_c$ (proportional to the length n of the message)
- transmitting a single message of n bytes takes time

$$T(n) = t_s + n \cdot t_c$$

Overview (continued)

- The **startup time** contains
  - the time to **construct** the message (inserting header with address and control information)
  - the **waiting time** required if the selected output link is currently busy
  - the **propagation time**, expressing the time between sending the first bit by the sender and receiving the first bit by the receiver
- For most networks, the startup time is significantly larger than the time required for an arithmetic operation
- In the following, we investigate the execution of communication operations on different networks
  Goal: derivation of **asymptotic running times**
  $\rightsquigarrow$ no exact timing formulas are derived

Specializations and Generalizations

The following facts can be used:

- If a communication operation can be executed in time x, then a specialization of the communication operation can also be executed in time x
- If a communication operation can be executed in time x, then a generalization of the communication operation takes at least time x

Section outline

Complete Graph Network

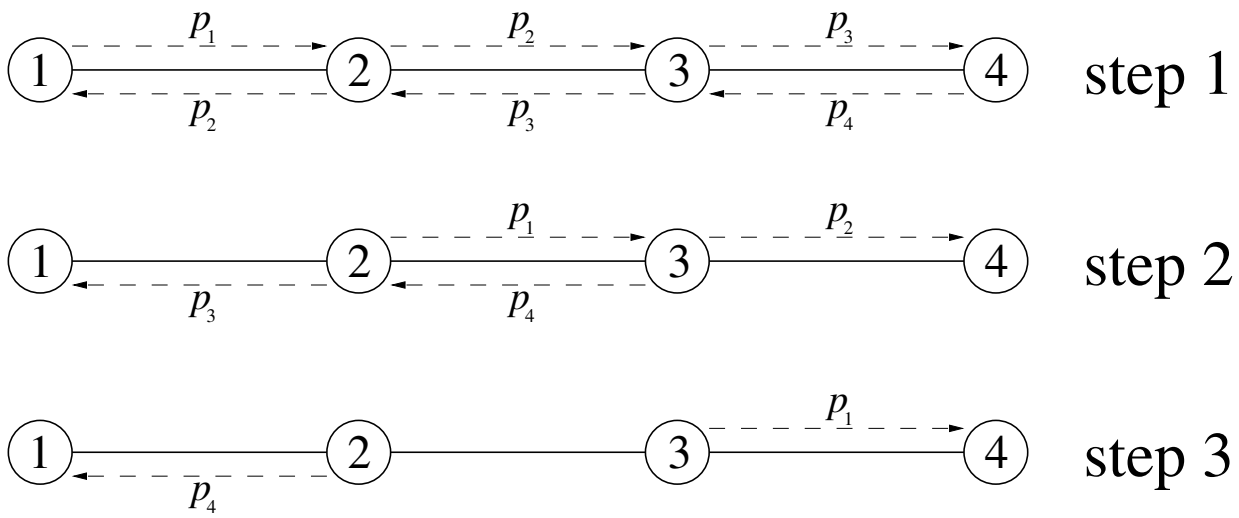
- **Single Gather** requires time $O(1)$:
  each node sends its message to the root node i
  node i can simultaneously receive all messages
  analogously: **Scatter**
- A **total exchange** can also be performed in time $O(1)$:
  each node sends out all its messages at the same time
  $\rightsquigarrow$ two messages are exchanged between two arbitrary nodes
  the corresponding link is bidirectional $\rightsquigarrow$ all exchanges require $O(1)$

Linear array Network

- $G = (V, E), V = \{1, \ldots, p\}, E = \{(i, i+1); 1 \leq i < p\}$
- **Single-broadcast**: the root process sends the message to its left and right neighbors; the neighbors forward the message step by step
  worst case: root process at the end of the linear array
  $\rightsquigarrow p - 1$ steps required
  best case: root process in the middle of the linear array
  $\rightsquigarrow \lceil p/2 \rceil$ steps required
- **multi-broadcast**:
  (a) 1st step: each node sends its **own message** to its two neighbors
  (b) 2nd step: each node $i \in \{2, \ldots, p-1\}$ receives the messages from node $i-1$ and $i+1$ and forwards them to nodes $i+1$ and $i-1$
  (c) step k: each node i with $k \leq i \leq p-1$ **receives** the message from node $i-k+1$ and forwards it to $i+1$
  each node i with $2 \leq i \leq p-k+1$ **receives** the message from node $i+k-1$ and forwards it to $i-1$

Linear array (continued)

illustration:



- The messages sent to the right (left) proceed in each step **one position** to the right (left)
  $\rightsquigarrow$ after $p-1$ steps, all messages have arrived at their destination
  $\rightsquigarrow$ running time $\Theta(p)$
- **Scatter** and **Gather** are **specializations** of multi-broadcast
  $\rightsquigarrow$ implementation in $p-1$ steps
  $\rightsquigarrow$ running time $\Theta(p)$

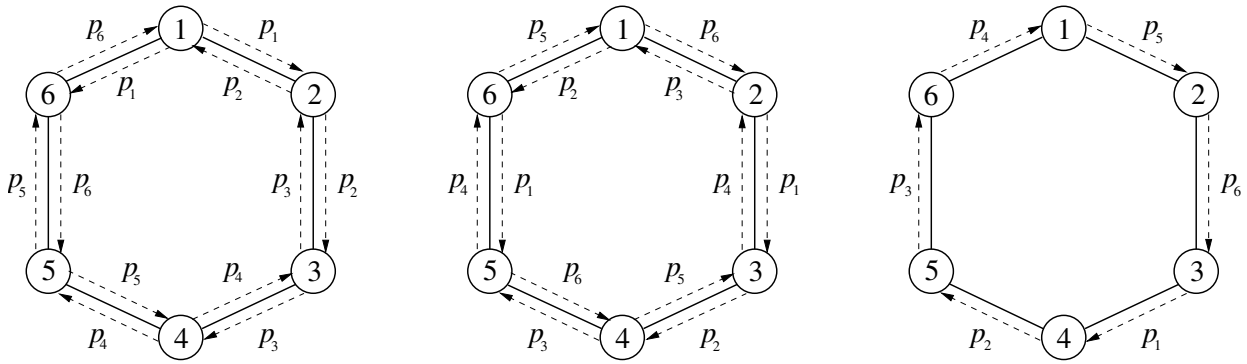
Linear array (continued)

total exchange: we consider an arbitrary edge $(k, k+1)$

- this edge splits the linear array in two subsets with k and $p-k$ nodes
- total exchange: each node in one of the two subsets sends a message to each node of the other subset
  $\rightsquigarrow k \cdot (p-k)$ messages must be transmitted over edge $(k, k+1)$
- for $k = p/2$: $p^2/4$ messages must be transmitted over a single link
  $\rightsquigarrow$ running time $\Theta(p^2)$

Ring network

- **single-broadcast**: $\lceil (p-1)/2 \rceil$ steps
- **multi-broadcast**: similar to a linear array network
  - step 1: each node sends its message in both directions
  - step k: $(2 \leq k \leq \lceil p - 1/2 \rceil)$
    each node sends the messages received in the opposite direction
    diameter $\lceil p/2 \rceil \rightsquigarrow$ running time $\lceil p/2 \rceil$
    illustration:



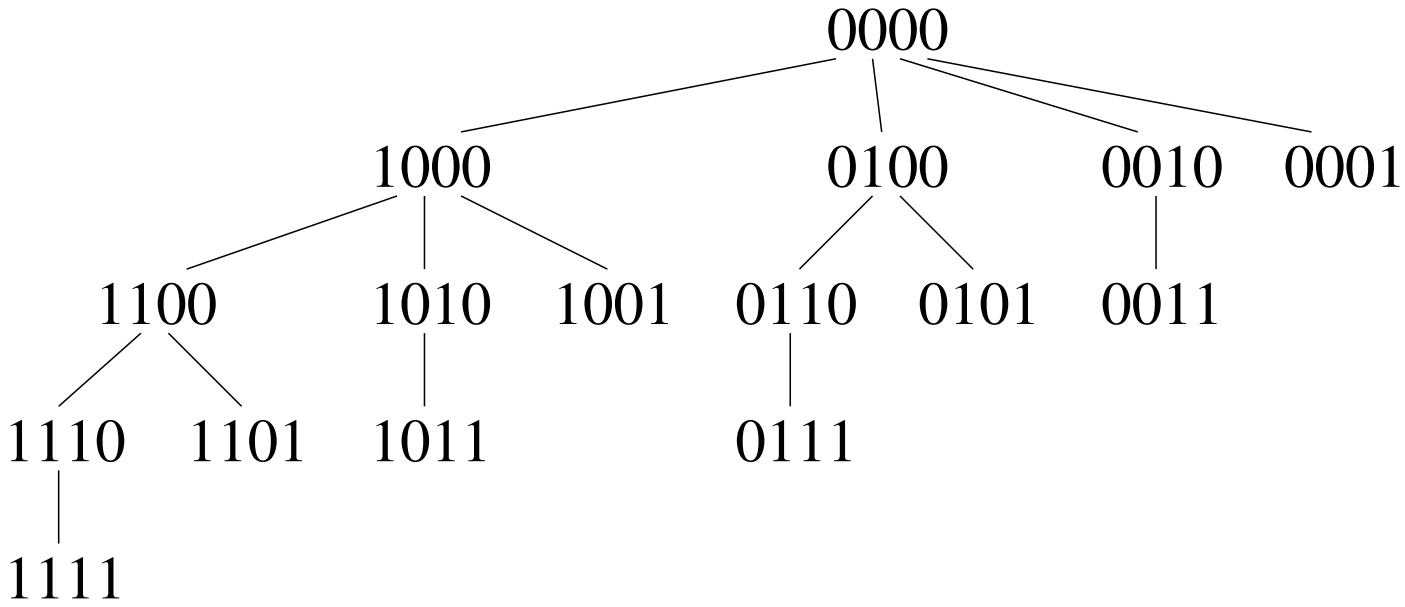
- **total exchange**:
  consider two nodes splitting the ring in two subsets with $p/2$ nodes each
  $\rightsquigarrow p^2/4$ messages must be transmitted over these two nodes (in each direction)
  $\rightsquigarrow p^2/8$ steps $\rightsquigarrow$ running time $\Theta(p^2)$

Section outline

3. Implementation of global communication operations
   - Communication operations on static networks
   - Communication operations on a hypercube network

Single-broadcast on a hypercube network

- We consider a hypercube network with d dimensions and $p = 2^d$ nodes
- Construction of a **spanning tree** for the network
- root at process $\alpha = 00 \cdots 0 = 0^d$
  the children in the tree are chosen by **inverting** one of the zero bits that are right to the rightmost unity bit
- **Illustration** for $d = 4$:



note: all child nodes differ from their present node in **exactly one bit position**

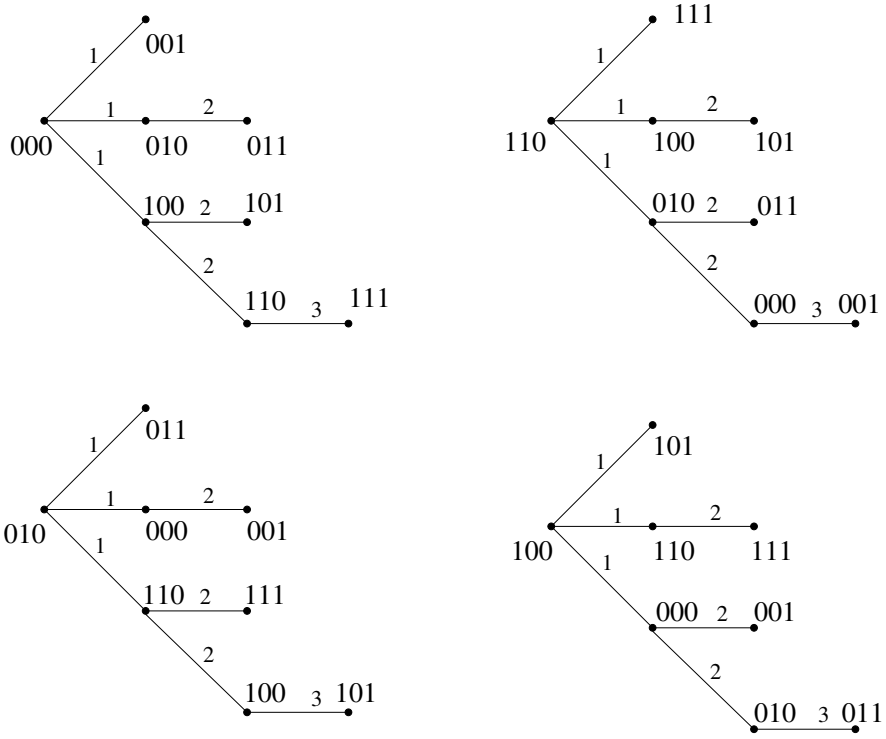
$\rightsquigarrow$ there is a **corresponding edge** in the hypercube network

Single-broadcast on a hypercube network (continued)

- The resulting spanning tree has **depth** d:
  consider an **arbitrary path** from a leaf to the root.
  Let (v_i, v_{i+1}) be an arbitrary edge on the path
  $\rightsquigarrow$ v_{i+1} contains one 1 less than v_i
  there are exactly d bit positions $\rightsquigarrow$ path has maximum length d
- Using an **arbitrary node** i as root, the spanning tree is constructed as follows:
  let $\oplus$ be the **bitwise exor operation**
  let T_0 be the spanning tree from above;
  The spanning tree T_i for root i results from T_0 by using $x \oplus i$ for each node x of T_0.
  Let (v, w) be an edge in T_0 $\rightsquigarrow$ v and w differ in **one bit position**
  $\rightsquigarrow$ $v \oplus i$ and $w \oplus i$ also differ in **one bit position**
  $\rightsquigarrow$ $(v \oplus i, w \oplus i)$ is an **edge of the hypercube network**
- **summary**: Single-broadcast in $d = \log p$ steps

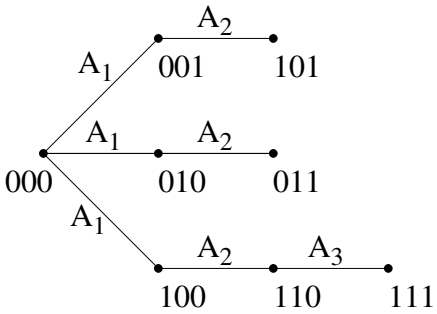
Multi-broadcast on a hypercube network

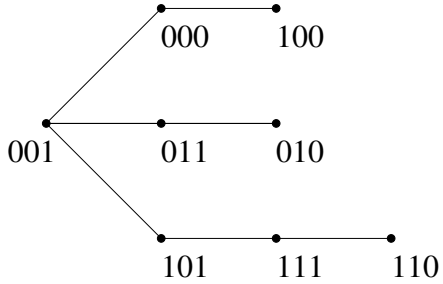
- Using the same spanning trees as for a single-broadcast leads to **collisions**: example: $d = 3$
  the spanning trees for root node 000 and 110 use the same edges $(010, 011)$ and $(100, 101)$, $\rightsquigarrow$ no simultaneous transmission possible
  Illustration:

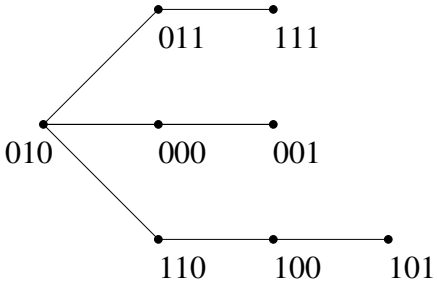


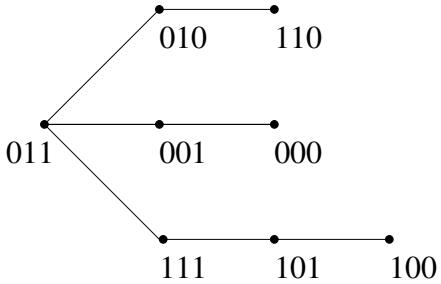
solution: use of alternative spanning trees

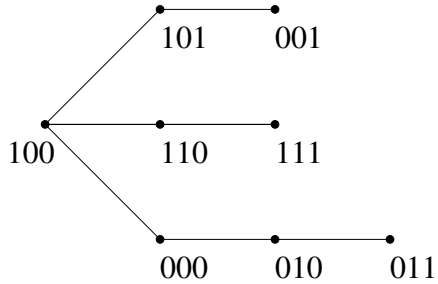
Spanning trees for multi-broadcast for hypercube

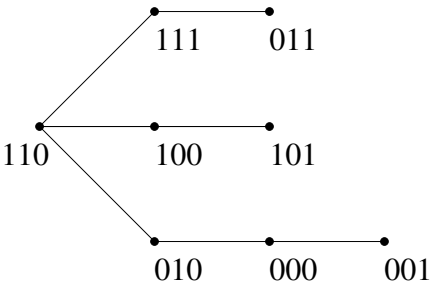


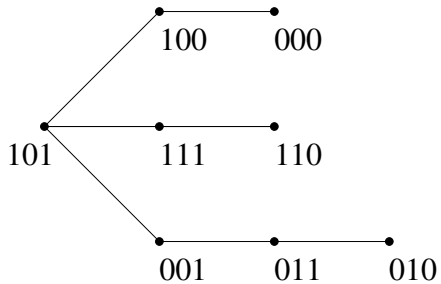


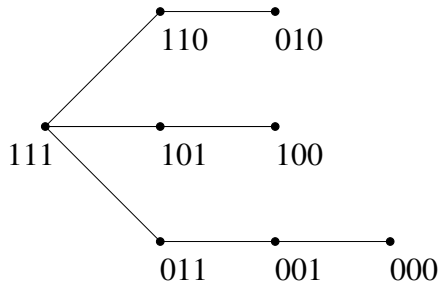












Construction of the spanning trees

- N_k = set of all nodes with bit representations containing k unity bits

$$\{00\cdots0\}N_1 \;\; N_2\cdots N_{(d-2)} \;\; N_{d-1}\{11\cdots1\}$$

$00\cdots0$ has position $n(00\cdots0) = 0$,
$11\cdots1$ has position $n(11\cdots1) = 2^d - 1$

- N_k is split into disjoint subsets $R_{k1}, \ldots, R_{kn_k}$
  R_{k_i} contains all nodes from N_k that can be transformed to each other by a **bit rotation to the left**
  The subsets R_{k_i} form equivalence classes of N_k
  The subsets R_{k_i} are ordered as follows:

$$\{00\ldots0\}R_{11}R_{21}\ldots R_{2n_2}\ldots R_{k1}\ldots R_{kn_k}\ldots R_{(d-2)1}\ldots R_{(d-2)n_{d-2}}R_{(d-1)1}\{11\ldots1\}$$

- Each node $t \in \{0,1\}^d$ gets a number $m(t)$ with $m(00\cdots0) = 0$ and $m(t) = 1 + [(n(t) - 1) \bmod d]$,
  i.e. the nodes are numbered in a **round-robin fashion** by $1\cdots d$.

Representation of the sets N_k

example: d = 4:

$$
\begin{array}{ll}
N_0 & \overset{0}{(0000)} \\
N_1 & \underbrace{\overset{1}{(0001)} \quad \overset{2}{(0010)} \quad \overset{3}{(0100)} \quad \overset{4}{(1000)}}_{R_{11}} \\
N_2 & \underbrace{\overset{1}{(0011)} \quad \overset{2}{(0110)} \quad \overset{3}{(1100)} \quad \overset{4}{(1001)}}_{R_{21}} \quad \underbrace{\overset{1}{(0101)} \quad \overset{2}{(1010)}}_{R_{22}} \\
N_3 & \underbrace{\overset{3}{(1101)} \quad \overset{4}{(1011)} \quad \overset{1}{(0111)} \quad \overset{2}{(1110)}}_{R_{31}} \\
N_4 & \overset{3}{(1111)}
\end{array}
$$

each node gets the number written above it

Definition of the node sets E_i

- We define $m+1$ node sets $E_0, E_1, \ldots, E_m$
- E_i is the set of all end nodes of edge sets A_i

$$E_0 = \{(00\cdots0)\}$$
$$E_i = \{t \in \{0,1\}^d \mid (i-1)d+1 \leq n(t) \leq i \cdot d\} \quad \text{for } 1 \leq i < m$$
$$E_m = \{t \in \{0,1\}^d \mid (m-1)d+1 \leq n(t) \leq 2^d-1\} \quad \text{with } m = \left\lceil \frac{2^d-1}{d} \right\rceil.$$

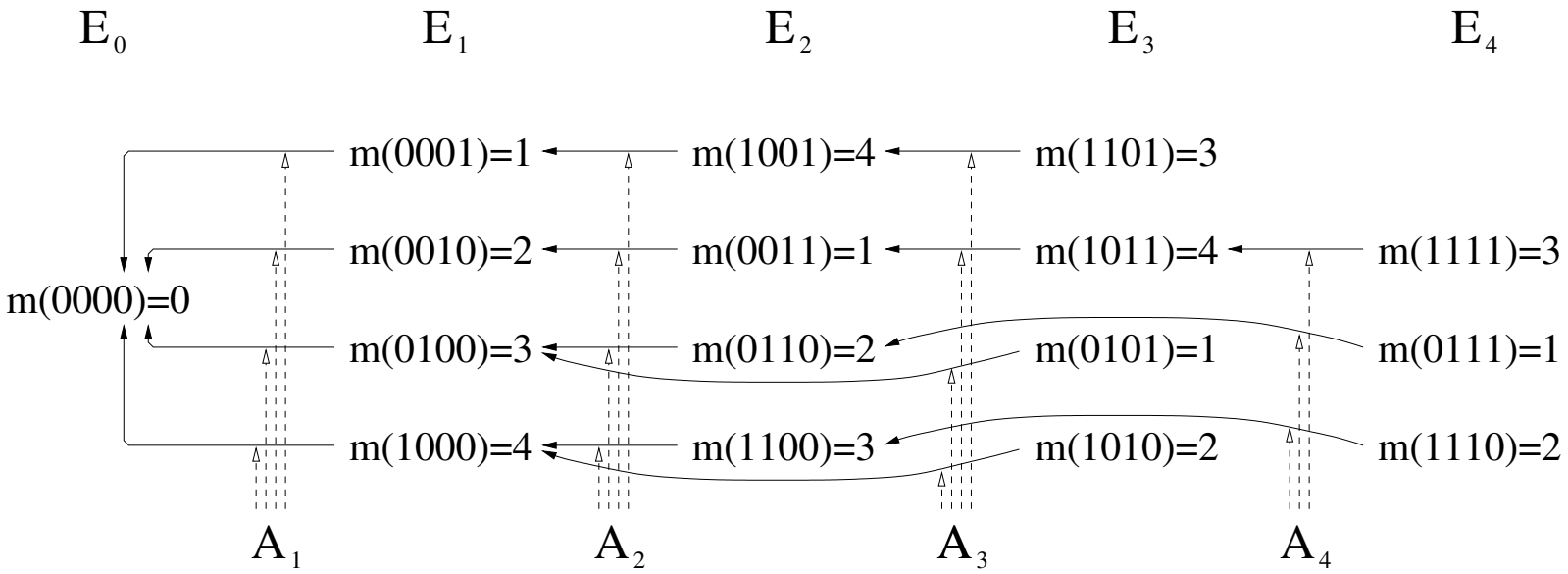
- For each set E_i the following holds:
  E_i contains d **contiguous nodes**
  $\rightsquigarrow$ all nodes in E_i have a different number $m(t) \in \{1, \ldots, d\}$

Definition of the edges sets A_i

- Each node $t \in E_i$ is connected with a node t' which results from t by **inverting** the bit at position $m(t)$ from the right;
  this bit is always a **unity bit** by construction
- **Special case**: if $m(11\cdots1) = d$, not the dth bit but the $(d-1)$th bit from the right is inverted, i.e., $((11\cdots1),(1011\cdots1)) \in A_m$, but $((11\cdots1),(011\cdots1)) \notin A_m$

Spanning tree for multi-broadcast

- we consider the case $d = 4$; the following spanning tree results



spanning tree with root $00\cdots0$;
the edge sets $A_i, i = 1, \cdots, 4$, of the different stages are indicated by dotted arrows

Total exchange on a hypercube network

- construction of a **recursive algorithm** with $2^d - 1$ steps
- $d = 1$: two nodes send a message to each other
- $d \to \phi + 1$: the hypercube of dimension $(d + 1)$ is split into two sub-hypercubes C_1 and C_2 with dimension (d)
- **Phase 1**: a total exchange is performed simultaneously in C_1 and C_2; each node of C_1 or C_2 exchanges messages with each other node of C_1 or C_2, respectively; this takes $2^d - 1$ steps
- **Phase 2**: each node in C_1 or C_2 sends its messages for all nodes in the other sub-hypercube to the corresponding node in this sub-hypercube
  Each node sends 2^d messages $\rightsquigarrow$ 2^d steps for this phase
- **Phase 3**: The messages received in phase 2 are **distributed** within C_1 or C_2 by a recursive use of the algorithm; this is similar to phase 1
  $\rightsquigarrow$ $2^d - 1$ steps for phase 3
- Phase 1 and 2 can be performed **simultaneously**, since different links of the hypercube network are used
  $\rightsquigarrow$ Phase 1 and 2 together require 2^d steps
- Phase 3 must be performed after phase 2
  $\rightsquigarrow$ total number of steps in $2^d + 2^d - 1 = 2^{d+1} - 1$
  $\rightsquigarrow$ overall running time $\Theta(p) = \Theta(2^d)$